An account of a Book. *Methodus* F*igurarum lineïs rectis & curvis comprehenſarum quadraturas determinandi Authore* J. Craige. 4to. Londini 1685..

THE great uſe of drawing the Tangents of Curve Lines, has made the moſt famous amongſt the Modern Mathematicians endeavour to find out General Methods of finding the Tangents of Curve Lines, as may be ſeen from the ſeveral ways invented by *Des Cartes*, *Monſieur Fermat*, *Sluſius*, Dr. *Barrow*, Dr. *Wallis*, *Tſchurnehuys*, and *Leibnitius*; But as yet none has attempted to invert this problem generally, that is, having the Tangent to find the Curve Line whoſe tangent it is. Therefore the Author of this Treatiſe perceiving that the doing of this would give a General Method of determinating the Quadrature of any Curvilinear ſpace, has laid down a rule for inverting *Sluſius* his method mentioned in the *Philoſophick Tranſactions Num.* 90. He has illuſtrated his Method of Quadratures by ſeveral Figures which have been already conſidered by Geometers. As for the Circle & Hyperbola, he aſſerts that their indefinite Quadratures are impoſſible, and therfore in theſe & ſuch like caſes, he expreſſes the Area by an infinite Series, which is eaſily done by his Method, except the Series conſiſt of irrational termes, for in theſe he has recourſe to *Leibnitius* his method of finding Tangents, where the Calculation will be more tedious. By his reſolving the Area of the Hyperbola into an infinite ſeries, he comes to the ſame expreſſion with that of *N. Mercator:* And in meaſuring the Zone of a Circle, his expreſſion falls in with that invented by Mr. *Iſaac Newton*, as Mr. *David Gregory* relates in his Treatise. He has ſubjoyned a Method of meaſuring the Curve Superficies made by the rotation of any Curve upon its *Axis*; with a ſmall Animadverſion on the Method of *Quadratrices*, publiſhed in the *Acta Lipſienſia Eruditorum* of *October*, 168 .

Since the Publication of this Treatiſe, the Authour is pleaſed to make the following Addition.

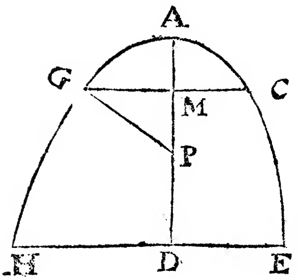
Additio ad Methodum Figurarum Quadraturas Determinandi. Autore *Johanne Craige.*

QUoniam omnium Figurarum Quadraturæ ex perfecta noſtri primi problematis Solutione determinantur; propterea utile judicabam nonnulla addere, quæ ſolutionem meam non modo plenius illuſtrant ſed omnino perficiunt: non adeo tamen ſunt obſcura, quin facile quiſquam in iſtiuſmodi rebus verſatus, ex iis quæ jam expoſui, omnia ſupplere poſſit. Problema ſic ſe habet. Data expreſſione Analytica lineæ inter ordinatam & Curvæ perpendicularem deſignatæ, Invenire æquationem naturam illius Curvæ definientem. Hoc problema tres caſus includit. 1. *Cum expreſſio iſtius lineæ talis eſt, qualis a vulgaribus tangentium Methodis exhibetur.* 2. *Cum ad ſimpliciorem reducitur, facta diviſione numeratoris & denominatoris per communem ſimplicem diviſorem.* 3. *Cum expreſſio ſit ſimpliciſſima, dividendo per diviſorem compoſitum. Duos priores caſus Regula, prout eam explicui, univerſaliter comprehendit; ſupereſt tantum ut oſtendam quo pacto tertium pariter caſum comprehendat.*

Poſtquam expreſſio data per y *multiplicatur, apponantur omnes termini qui ſub maximo continentur (Terminorum magnitudinem e dimenſionibus quantitatis* y *menſurans) & connectantur ſigno affirmativo vel negativo, prout libuerit; adæquentur omnes illi termini (prius in coefficientes incognitas multiplicati) Quadrato quantitatis per* x *deſignatæ: eritq; inde reſultans æquatio quæſita, vel quæſitam includet; & determinationes coefficientium terminos æquationem conſtituentes a reliquis diſtinguent.*

Sit in appoſito ſchemate abſciſſa AM$=$y, *ordinata* MC$=$z, *& Curvæ* ACE *proprietas* $z2=\frac{a2y2+y4}{p2}$, *& invenienda ſit quadratura Areæ a lineis rectis & illa Curva comprehenſæ. Quærenda eſt alia Curva* A G H, *in qua* P M$=\sqrt{\frac{a2y2+y4}{p2}}=$z; *ubi* P G *Curvæ quæſitæ perpendicularem, &* MG$=$x *illius ordinatam denotat. Cumq; hæc expreſſio lineæ* PM *in* y *multiplicata contineat ſextam quantitatis* y *dimenſionem, ideo appono omnes terminos ſub illa ſexta dimenſione contentos, unde reſultans æquatio eſt.*

$$\frac{na6+ma5y+la4y2+ha3y3+ka2y4+gay5+fy6}{p\,2}=x4.$$



Ex hac æquatione invenio valorem Lineæ P M. *quem comparo cum valore dato, unde*

$$P\,M=\frac{ma5+2la4y+3ha3y2+4ka2y3+5gay4+6fy5}{4p\sqrt{na6+ma5y+la4y2+ha3y3+ka2y4+gay5+fy6}}$$

$=\sqrt{\frac{a2y2+y4}{pp}}$. *auferantur fractiones & Signa radicalia, & determinentur coefficientes* n, m, l &c: (*ut in* prob: 2 *tractatus noſtri*) *rejectis iis quarum determinationes abſurdum involvunt: & cæteræ, in quibus nihil tale contingit, æquationem conſtituent. Sic in exemplo propoſito, erit* f$=\frac{4}{9}$, k$=\frac{12}{9}$, l$=\frac{12}{9}$, n$=\frac{4}{9}$. *ſed dum* g *determino, invenio* 240g$=$144g, *quod abſurdum involvit; & ſic pro* h *comparatio erit* 16h$=$16h *unde nullus illius valor, & pro* m *erit* 4 m$=$4 m, *quod itidem eſt abſurdum: quapropter termini a quantitatibus* g, h, m *affecti ad æquationem non pertinebunt; unde reliqui a literis* n, l, k, f *affecti æquationem naturam Curvæ definientem conſtituent. ſc.* $\frac{4a6+12a4y2+12a2y4+4y6}{9p2}$ $=$x4,

$=x^4$, *adeoq;* AMC$=\sqrt{\frac{a^6+3a^4y^2+3a^2y^4+y^6}{6p^2}}=\frac{x^2}{2}$. *Exem.* 2

Sit Curvæ Lineæ A C E *talis proprietas* $z^2=\frac{qy^2+y^3}{q}$ *& inveniendа ſit Quadratura ſpatii* A M C. *Quærenda eſt Curva* AGH *in qua ſit* P M$=\sqrt{\frac{qy^2+y^3}{p}}=Z$ *& quoniam hic valor in* y *multiplicatus continet quintam quantitatis* y *dimenſionem, apponantur omnes termini ſub illa quinta dimenſione, & æquentur quadrato quantitatis per* x *deſignatæ; unde æquatio reſultans eſt.*

$\frac{nq^5+mq^4y+lq^3y^2+kq^2y^3+hqy^4+fy^5}{p}=x^4$. *atq; ſola coefficientis* (m) *determinatio abſurdum involvet, eruntq; reliquæ,* $n=\frac{64}{225}$, $l=-\frac{16}{15}$, $k=-\frac{16}{45}$, $h=\frac{16}{15}$, $f=\frac{16}{25}$, *unde æquatio curvam quæſitam definiens eſt.*

$\frac{64q^5}{225p}-\frac{16q^3y^2}{15p}-\frac{16q^2y^3}{45p}+\frac{16qy^4}{15p}+\frac{16y^5}{25p}=x^4$ *adeoq;*

AMC$=\sqrt{\frac{16q^5}{225p}-\frac{4q^3y^2}{15p}-\frac{4q^2y^3}{45p}+\frac{4qy^4}{15p}+\frac{4y^5}{25p}}=\frac{xx}{2}$

Exem. 3. *Inuenienda ſit Quadratura ſpatii* AMC, *definita natura Curvæ* ACE *hac Æquatione* $z^2=\frac{a^3}{4y+4a}$. *Quæratur alia Curva* AGH, *in qua* PM$=\sqrt{\frac{a^3}{4y+4a}}=z$. *Ex præmiſſis conſtat Æquationem primam fore* $\frac{na^3y^2+ma^4y+16a^5}{4a+4y}=4x$ *& determinationes Coefficientium* $n=16$, $m=32$, $l=16$. *Quibus ſubſtitutis, erit æquatio* $\frac{16a^3y^2+32a^4y+16a^5}{4y+4a}=x^4$ $=4a^3y+4a^4$: *adeoq;* AMC $=\sqrt{a^3y+a^4}=\frac{1}{2}x^2$.

Notatu digniſſimum eſt, has tres (ſicut infinitas alias) Quadraturas abſciſſæ AM (*ſeu* y) *non convenire. Quoniam in iſtiusmodi*

modi

modi Figuris, ſimpliciſſima Areæ expreſſio huic portioni non reſpondet: attamen Quadratura abſciſſæ conveniens exinde parvo labore deducitur. Ut in Exem: 3. ubi Area eſt $\sqrt{a3y+a4}$; fiat $y=0$, & erit Area $\sqrt{a4}=a2$, & ſubducatur hæc ex generali; proveniet Quadratura portionis abſciſſæ reſpondentis, ſc. $\sqrt{a3y+a4}-a2$. Quam obſervatiunculam mihi primus ſignificavit Vir celeberrimus D. Iſaacus Newton.

Tentetur jam idem proceſſus in Circulo ACE, cujus diameter ſit r, ac proinde $Z=\sqrt{ry-y2}$, Quærenda eſt Curva AGH in qua $PM=\sqrt{ry-y2}=z$, ſed ex dictis conſtat æquationem primam fore $nr4+mr3y+fr2y2+hry3-ky4=x4$: & ſingulæ coefficientium determinationes erunt impoſſibiles; adeoq; nulla datur Curva AGH in qua $PM=\sqrt{ry-y2}$, ac proinde Circuli Quadratura indefinita eſt impoſſibilis. Fieri tamen poteſt ut ſit aliqua hujuſmodi Curva AGH, ſed ex earum numero, quas poſt Carteſium Mechanicas Geometræ communiter appellant: ſed quia harum uſus non libenter admittunt Mathematici, præſtat hujuſmodi Quadraturas per ſeries infinitas exhibere.

Benevole Lector

Ob inopiam Typorum Numeralium minuſculorum, qui ad deſignandas quantitatum poteſtates ſupra Symbola dextrorſum apponi ſolent, feſtinante prælo, Typographus paulo majoribus uſus eſt in eadem linea immediate ſequentibus; ubicunq; itaq; offenderis a3, *vel* x2, *&c. cubum vel quadratum, &c.* e *quantitate, cui ſuffigitur numerus, intelligas.*

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